

## **An Evolved Psychological Structure for Dealing, Food Sharing, and Mathematical Division**

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### **Abstract**

Research has shown that very young children have the ability to solve difficult division problems before formal school-based instruction. This is accomplished by solving sharing-based division word problems by an interesting act known as 'dealing'. The theory set forth in this manuscript is that children's ability to solve sharing-based division problems is due to an evolved psychological structure expressed as a dealing action. This structure may have been shaped by an ancient ecology, which pressured human ancestors to cooperatively share food in order to survive, and contributed to the emergence of formal mathematical division. This theory is supported by evidence of a specialized action for dealing to solve sharing-based division problems, a pancultural ability for young children to solve the problems, children's lack of consciousness while solving the problems, development from dealing to a mental model, evidence of evolved quantitative abilities in humans and animals, food sharing behaviors in humans and capuchin monkeys, and egalitarian abilities. The discussion section includes: a possible scenario for the development of the structure; an illustration of how the structure can be used in the elementary classroom; and empirical questions which may lead to a better understanding of food sharing habits, ecology, and sociology of early humans.

**Keywords:** mathematical division, evolved psychological structure, food sharing, quantitative abilities, capuchin monkeys, dealing, problem solving

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### **Introduction**

Research over the past three decades has shown that children can solve difficult division problems before second grade without formal educational instruction (Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993; Carpenter, Fennema, Franke, Levi, & Empson, 1999; Clements & Lean, 1988; Davis &

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Pitkethly, 1990; Empson, 2001; Frydman & Bryant, 1988; Hunting & Davis, 1991; Hunting & Sharpley, 1988). The division problems that young children can solve are not in the form of or with the use of the traditional algorithm or symbolic notation; but particular division word problems that are solved by an act that resembled sharing and is called dealing (Correa, Nunes, & Bryant, 1998; Davis & Pitkethly, 1990; Hunting & Davis, 1991). Researchers are puzzled by this interesting phenomenon and attribute the ability to informal knowledge (Mack, 1998), equivalence-reasoning strategies (Empson, 2001), and unlearned strategies (Pepper, 1991). It is believed that this unexplained ability may be the basis for learning formal division (Carpenter, Fennema, & Franke, 1996) and could explain other mathematical abilities (Pepper & Hunting, 1998; Squire & Bryant, 2002a).

Multiple conjectures for the origin of the ability to divide have been proposed (Pepper & Hunting, 1998). Frydman and Bryant (1988) and Hunting and Davis (1991) believe that it is learned from others, Correa et al. (1998) state that it is an action schema tied to sharing, Riess (1955) claims it arises developmentally through experiences, and Ansari (2008) states some mathematical abilities may have been shaped by natural selection. These conjectures are the furthest explanations for why children can solve difficult sharing-based division problems before second grade without formal educational instruction. This surprising void in the literature is probably due to the necessary interdisciplinary nature of the argument, and the lack of evidence for how children solve sharing-based division problems. Fortunately, new evidence on the origin of evolved psychological structures and how children solve sharing-based division problems has shed light on the topic; therefore, a coherent theory can now be formed. The theory set forth in this manuscript is that children's ability to solve sharing-based division problems is an expression of an evolved psychological structure for dealing. This structure may have been shaped by an ancient ecology, which pressured human ancestors to cooperatively share food in order to survive, and contributed to the emergence of formal mathematical division.

### **Theoretical Framework: Evidence for an Evolved Psychological Number Structure**

#### *Implicit Knowledge and Number Sense*

Evolved psychological structures are knowledge, abilities, or competencies (Geary, 2005) rooted in specialized brain circuits. The structures are believed to be encoded in our genome, inherited much like physical evolved structures (i.e., adaptive traits), and shaped by recurring past social or ecological problems. The best evidence that these structures exist is specific behaviors, which occur when extant organisms face particular social or ecological cues. These structures extract one type of specific information from the environment through the senses, and

respond quickly, automatically, and unconsciously. Humans often have difficulty in accurately describing what promoted the response (Geary, 2005). It has been hypothesized that these structures were, and sometimes continue to be, advantageous to the individual and the population. It is further hypothesized that many of these structures have been co-opted by culture and built upon to form academic disciplines and concepts (Ansari, 2008; Geary, 1995, 2005).

Many propose that humans have an evolved psychological structure for number (number structure hereafter) which enables an individual to categorize the world in terms of numerosities and their relationships (Ansari, 2008; Butterworth, 1999; De Cruz, 2006; Dehaene, 1997; Dehaene, Dehaene-Lambertz, & Cohen, 1998; Geary, 1995). This structure was probably important for survival and was involved in foraging decisions or determining numerosities of predators (Ansari, 2008). Specifically, this structure gives one number sense, or the ability, within limits, to individuate the numerosities in a collection, instantaneously count or estimate numerosities in a collection (called subitizing), match the numerosities of one collection based on one-to-one correspondences between count-words and the objects being counted, explain which of two collections is larger, enumerate collections, and recognize changes in numerosities within collections (Butterworth, 1999; Geary, 1995).

### *Number Sense in Infants*

Several studies have tested infants' number sense and found that they are sensitive to numerosities (Starkey, 1992; Starkey, Spelke, & Gelman, 1991), recognize visual differences in continuous (e.g., length and area) and discontinuous (e.g., number of dots) variables (McCrink & Wynn, 2004), recognize differences between auditory syllables and tones (Bijeljac-Babic, Bertoni, & Mehler, 1993; Starkey & Gelman, 1983; vanMarle & Wynn, 2002) and can conduct simple arithmetic (McCrink & Wynn, 2004; Wynn, 1992). Newborns can recognize numerosities up to three within the first weeks of life (Antell & Keating, 1983) and can represent numerosities up to three or four exactly, but fail at recognizing differences between higher numerosities (De Cruz, 2006). Since infants have the ability to disregard the modality of presentation (i.e., visually and auditorily), yet hold a sensitivity to numerosities, it is believed that the representation is abstract (Starkey & Gelman, 1983). These abilities are believed to appear pan-culturally (Geary, 1995).

### *Neurological Evidence*

Adult, infant, and animal neuroimaging studies have provided additional evidence for a number structure (De Cruz, 2006). Overwhelming evidence shows that arithmetic facts and calculation procedures are stored in the parietal lobe (Rickard et al., 2000), and numerous adult studies have determined the

intraparietal sulcus (IPS) may be the center of number processing and visuospatial attention (Cohen Kadosh et al., 2007; Dehaene, Piazza, Pinel, & Cohen, 2003; Fias, Lammertyn, Reynvoet, Dupont, & Orban, 2003). Eger, Sterzer, Russ, Giraud, and Kleinschmidt (2003) found that the IPS is activated when subjects are presented with both visual and auditory Arabic digits, whereas letters presented in the same fashions did not elicit a response. Dehaene, Spelke, Pinel, Stanescu, and Tsivkin (1999) found that the IPS, and not cerebral circuits associated with language, was activated when subjects were asked to use approximate calculations to answer various types of mathematics problems. It is believed that these neural circuits may contain neurons that are sensitive to numerosities. Nieder and Miller (2004) found that when trained rhesus monkeys were shown dots of various sizes, shapes, and colors, specific individual neurons were only activated when changes to the number of dots were presented. These individual neurons were activated by particular numerosities, and decreased in activation as the numerosities increased or decreased. Certain cells in the parietal-occipital cortex of cats behaved similarly, and selectively responded to tactile, auditory, and visual numerosities (Thompson, Mayers, Robertson, & Patterson, 1970).

### *Number Sense Deficits*

Since the number structure is biological and inherited, it may fail to develop normally in some and can be impaired by disease and brain trauma in others (Butterworth, 1999). Developmental disorders include developmental dyscalculia, which affects children with normal intelligence and causes difficulty in calculation and mental arithmetic (Price, Holloway, Rasanen, Vesterinen, & Ansari, 2008). The IPS activity in children with developmental dyscalculia is disrupted during certain mathematical tasks (Fias et al., 2003), and is absent in others (Price et al., 2007). Recent studies show that children with developmental dyscalculia were found to have less grey matter in the left (Isaacs, Edmonds, Lucas, & Gadian, 2001), and right IPS (Rotzer et al., 2008). Lesions caused by disease to the parietal lobes can impair internal representations of numerosities (Takayama, Sugishita, Akiguchi, & Kimura, 1994; Warrington, 1982) and number processing impairments, which render one without an ability to comprehend, produce, and calculate numbers (Dehaene et al., 1998). Examples of extreme impairment due to damage to this region of the brain have left subjects without the ability to subitize two dots; determine if the numeral 5 is larger or smaller than the numeral 10; or remember one's home address, age, or shoe size (Butterworth, 1999). Virtual lesions can be created by using transcranial magnetic stimulation to stimulate the right parietal lobe, which in turn, hinders automatic processing in adults (Cohen Kadosh et al., 2007).

### *Numerosities in Animals*

If a number structure is encoded in our genome, we may be able to trace it back through our evolutionary history and find evidence of it in extant organisms (Butterworth, 1999). Decades of animal studies show that certain organisms have a latent capacity to perform certain numerical tasks (Butterworth, 1999; Gallistel, 1990). Evidence of sensitivity to numerosities among other number sense has been found in the African grey parrot (Pepperberg, 1987), domestic cat (Thompson et al., 1970), common chimpanzee (Boysen, 1993), rhesus monkey (Hauser, Carey, & Hauser, 2000), lion (McComb, Packer, & Pusey, 1994), and red-back salamander (Uller, Jaeger, Guidry, & Martin, 2003). Other mathematical abilities have been studied in invertebrates such as the honey bee and digger wasp (Gallistel, 1990; Gould, 1986).

There are evolutionary adaptive advantages for numerical abilities in animals. Environmental characteristics that contain information, and often vital information, can exert selective pressures on animal nervous systems (De Cruz, 2006). Therefore, a number structure could reduce multiple complex inputs, like objects or events in time and space, to simple numerical relationships (De Cruz, 2006). For example, it is advantageous for animals to have an ability to compare two collections of food in environments where food is scarce, and to determine which is greater. This ability would improve an organism's fitness in its environment, therefore, improving its chances for survival and reproduction. Animal studies have shown that some organisms use numerical cues to make foraging decisions (Hauser et al., 2000; Uller et al., 2003), some compare the number of individuals in their group to others in order to determine whether to attack the group (McComb et al., 1994), and some use auditory numerosities to communicate to others within their group (Boesch, 1996).

## **Evolved Psychological Structure for Dealing**

### *Dealing and Children*

Since the capacity to divide through sharing manifests at an early age, the ability seems to be automatic and unconscious (Davis & Pitkethly, 1990), and children are proficient at it (Frydman & Bryant, 1988; Squire & Bryant, 2002b); the ability may be attributed to an evolved psychological structure. Evolved psychological structures, like the number structure, can be expressed through abilities, knowledge, or competencies (Geary, 2005). The ability which is expressed through this proposed evolved psychological structure is called dealing. Davis and Pitkethly (1990) define dealing as a "...cyclic distribution of discrete objects (regarded as identical), with the same number distributed to each place on each round of the cycle until there are none left [p. 145]." Children use this ability, which

resembles the dealing of playing cards, to solve sharing-based division problems (Correa et al., 1998; Davis & Pitkethly, 1990; Hunting & Davis, 1991).

If an evolved psychological structure is encoded in our genome and inherited, then the ability to solve sharing-based division problems through dealing should be found panculturally. This seems to be the case. For example, Hunting and Sharpley (1988) found that 206 English-speaking Australian children who did not have traditional educational instruction in division, successfully allocated 12 items to three dolls 60% of the time, and 75 four and five year olds allocated 12 crackers to two dolls 77% of the time. Correa et al. (1998) observed the dealing ability in five year old English-speaking children in England when solving sharing-based division problems. Empson (1999) and Mack (1993) found that first grade and fifth grade American children successfully used the ability to solve sharing-based division problems in a classroom setting.

Children in isolated tribes, where there is little contact with other groups which could culturally transmit the ability and knowledge, can successfully solve sharing-based division problems through dealing (Butterworth, Reeve, Reynolds, & Lloyd, 2008). Researchers studied 45 children between the ages of four and seven. Monolingual speaking children of two Australian languages (Warlpiri and Anindilyakwa), that lacked counting words, were tested on numerical concepts against English speaking indigenous Australian children. An enumeration task was used to test number sense understanding which included the sharing of multiple, equal-sized, three-centimeter disks. The children were asked a division problem in which children shared the disks among three toy bears. In four randomly-ordered trials, children were given six, seven, nine, and ten disks. The results of the experiment showed that nearly all the children tested were able to successfully share six or nine items among three bears. Similarly, both groups of children were less successful sharing seven and ten disks among the bears. The researchers concluded that the findings could be explained by an inherent mathematical ability (Butterworth et al., 2008).

The ability to solve sharing-based division problems by dealing is automatic, unconscious, and difficult for the individual to accurately describe what promoted the response (Davis & Pitkethly, 1990; Frydman & Bryant, 1988). The social cue, which is often and successfully used in the classroom setting (e.g., Empson, 1999, 2001; Empson, Junk, Dominguez, & Turner, 2005), occurs when one is asked to share  $x$  items to  $y$  individuals (Correa et al., 1998). As with other evolved psychological structures which are automatic and unconscious, children only have an implicit knowledge of when to deal. The unconscious act of dealing is not the same as the conscious understanding of division. In other words, children do not understand the relationship between the dividend, divisor, and quotient (Correa et al., 1998). In fact, Davis and Pitkethly (1990) discovered that when preschool children used dealing to solve sharing-based division problems, they were often not aware that they dealt the same amount to each individual. The children had to count

the shared items that each individual received, and used counting as an essential part of solving the problem as opposed to a method of checking. Mathematics educators are aware that this and other automatic, unconscious abilities may actually interfere with an understanding of mathematics. Therefore, these abilities, like any other misconceptions/alternative conceptions, are to be used as starting points to teaching mathematical concepts.

### *From Dealing to Formal Mathematical Division*

Some evolved cognitive structures are co-opted by culture, and built upon to create formal academic topics and academic disciplines (Geary, 1995, 2005). The work of Squire and Bryant (2003; also see 2002a, 2002b) seems to demonstrate that dealing is a primitive form of formal mathematical division; that is, children's implicit knowledge moves beyond the simpler physical dealing of manipulatives (e.g., crackers, disks, food) to a model of active mental dealing. This significant research was based on the distinction of the two types of division word problems which appear in real life scenarios: partitive and quotitive. Partitive division is a problem type in which one evenly shares the number of items in a collection (the dividend) to a known set of subcollections (the divisor), which yields a quotient (Greer, 1992). An example of a partitive division problem is: Lakeisha has 12 buttons and wants to evenly share the buttons among three friends. How many buttons does each friend get? Quotitive division is a problem type in which one determines how many subcollections (the quotient) of a fixed size (the divisor) can be created from a collection (the dividend) (Greer, 1992). An example of a quotitive division problem is: Lakeisha has 12 buttons and wants to evenly share the buttons with her friends. If each friend receives three buttons, how many friends received buttons?

Squire and Bryant (2003) devised an ingenious technique in which students could solve problems by either grouping by the divisor or quotient. If children had an understanding of the relationship between the divisor and the quotient, there should be no difference found between the problem types and children's use of grouping to solve the problem. This is because each of these problems has two separate correct outcomes and the children could correctly solve the problem by either grouping by the divisor or quotient. One hundred twenty-nine, five to eight years old children were presented with drawings of collections arranged in a grid pattern and were read either partitive or quotitive division problems. It was discovered that the children were more successful at solving partitive division problems that are grouped by the divisor versus the quotient, and the opposite when solving quotitive division problems.

Squire and Bryant's (2003) study shows that children have a restricted model of sharing that mirrors how they would solve the problem by dealing. In other words, children solved the problems by mentally dealing, and not by an understanding of the relationship between the divisor and the quotient. The

internalization of dealing provides evidence that sharing-based division problems can develop into a mental model. This is significant because models, which quantitatively represent the behavior of a system, are formulated by mathematicians by reducing systems in the natural world. This study furthers the theory that action schemas, such as dealing, are the origin of children's understanding of mathematics operations (Correa et al., 1998).

### *Food Sharing and the Evolution of Ape to Man*

An evolved psychological structure for dealing is supported by evidence of a specialized action by a particular cue, a pancultural ability in young children, children's lack of consciousness while solving problems, children's development from dealing to a mental model, and evidence of a number structure in humans and animals. These characteristics fit the description of an evolved psychological structure (Buss, 2004). If an evolved psychological structure for dealing does exist, and evolved psychological structures are believed to be shaped by past environments; then we must look into the past for a situation in which dealing, sharing, or a related ability solved an ongoing problem. Since humans are the only extant species that have all of the aforementioned characteristics of the evolved psychological structure for dealing, the structure had to come about during the evolution of ape to man. Anthropologists and paleontologists have listed many factors that may have contributed to the evolution of ape to man, and the theme of sharing, specifically food sharing, is prevalent in human evolution literature. Food sharing is found in every proposed theory for the evolution of ape to man as either a primary (Isaac, 1978; McGrew & Feistner 1992; Tanner, 1987) or secondary factor (Lovejoy, 1981; Parker & Gibson, 1979). Each theory contends that humans evolved in an egalitarian society where food was equally shared. Evidence of food sharing in early man includes large collections of tools alongside tool-scarred, mammal bones (Isaac, 1978); the study of food sharing behaviors of modern hunter-gathers; and food sharing behaviors in animals.

The reason that the sharing of food is a theme in the evolution of ape to man is because the ability to share was a solution to a problem that occurred during this transition. Food sharing would have been tremendously advantageous because of the precarious nature of hunting. Hunting is an activity that does not always yield a kill, and modern hunter-gather societies, which provide the best evidence for the behavior of early man (Knauff, 1993), often return with no food to feed the group. For example, the modern hunter-gathers, the Ache of Paraguay, have a 40% chance of returning from a hunt empty-handed (Hill & Hawkes, 1983). However, when a hunter does bring back food; meat for example, it is best to share the meat because one can only eat so much of it. If the meat is not shared, it will spoil. Additionally, it often takes multiple hunters, working cooperatively, to hunt and kill game. Therefore, it is reasonable that hunters would share their food when they were successful. Food sharing is pervasive in modern hunter-gathers societies, has been

shown to reduce the chances of risk from starvation (Kaplan & Hill, 1985), and its benefits align with optimality analyses models (Cosmides & Tooby, 1994).

### *Food Sharing in Nonhuman Primates*

Evolved structures, whether they are physical or psychological, or occur gradually or suddenly in deep time, can often be traced through homologous traits in common ancestors. Most food sharing behaviors seem to follow phylogenetically predictable lines (McGrew, 1992), and great ape and human infants seem to share homologous psychological structures (Bard, 1990; Mathieu, 1982). However, these structures are often exhibited as different abilities and it is difficult to ascertain if the structures are homologous. For example, both children and apes experiment with object-force relations (Bard, 1990). Children experiment by throwing toys at a wall with varying forces while observing outcomes and internalizing relationships and models, just as young apes playfully hit one another with varying forces while observing the other's reaction (Bard, 1990). These seemingly interesting and pleasurable actions allow both humans and primates to understand the physical world, and how to interact with it.

Some food sharing abilities are believed to be an expression of an evolved psychological structure, and food sharing behaviors have been observed in nonhuman primates and non-primates, but to various degrees and kinds. Lions, wolves, hyenas, mongooses, birds, and vampire bats are non-primates which participate in food sharing. The sharing does not use a dealing action, is not habitual or active, and usually takes the form of tolerate theft or scourging. Habitual food sharing has been observed in at least four primates: capuchins, chimpanzees, callitrichids, and orangutans. Capuchins actively share food with a partner, and seem to share food as opposed to more selfish options (de Waal, Leimgruber, & Greenberg, 2008). They have been observed actively sharing stone tools (Westergaard & Suomi, 1997) and artificial tokens with other capuchins (Westergaard, Liv, Chavanne, & Suomi, 1998). This has been observed in captivity and without the use of a dealing action. Chimpanzees often share food by tolerated scrounging, by which the parent allows the offspring to take the food leftovers (McGrew, 1992). Although they do not seem to actively share edible food with each other, they do actively transfer nonfood items (Celli, Tomonaga, Udono, Teramoto, & Nagano, 2006). This has been observed in captivity through the active sharing of plastic pins, metal brushes, plastic spoons, other artificial items without a dealing action. Callitrichid sharing is active, without prompt, and occurs most often between parents and older siblings to infants, and less often between family members and the recipient mother or juvenile (McGrew, 1992). Orangutan mothers actively, but rarely, transfer food to their young, usually by breaking off a piece and putting it in the infant's mouth (Bard, 1990).

It is believed that the active sharing of food does not occur as often in nonhuman primates as it does in humans because sharing is less important for survival (McGrew, 1992), and there seems to be little need for it (de Waal, 1996). Interestingly, besides chimpanzees, capuchins are the only nonhuman primates to prey on large vertebrates (Rose, 1997), and meat is the only food that has been observed to be actively shared between adults in a naturalistic setting (Perry & Rose, 1994; Rose, 1997; Rose & Fedigan, 1995).

In order to claim an extant nonhuman primate has an evolved psychological structure for dealing, two criteria must be met: a) the ability to deal food or a homologous ability, and b) a social or ecological cue which elicits the dealing action or homologous action. Of the four primates which habitually share food, the capuchin monkey best fits the criteria. The capuchin monkey is a new world ape of the genus *Cebus*, and shared a common ancestor with humans that lived over 25 million years ago. The monkey has been observed raiding the nest of coati; a member of the raccoon family (Perry & Rose, 1994). Monkeys steal young coati pups and eat them alive, but they also tear off pieces of the pups and distribute them to begging monkeys within the group (de Waal, 1996). This observation is significant because the monkey actively offers food through an action similar to dealing, to multiple monkeys. However, researchers did not report that the distribution was cyclical or that it was repeated until there was no meat left. It is rare for non-human primates to share, especially by actively offering food with other adults and those outside of the parent-offspring context (de Waal, 2000).

Simply tearing off pieces of carcass and distributing meat is not enough for the action to be related to the evolved psychological structure for dealing. There has to be either a social or ecological cue which elicits the homologous action; a cue similar to the oral cue used by individuals (e.g., teachers, researchers) so children can solve sharing-based division problems. It appears that capuchins will only actively share meat with beggars who use specific posture or vocalization, and interest in the meat (Brown, Almond, & van Bergen, 2004; Perry & Rose, 1994). The benefactors will actively set down the meat in front of the beggar (de Waal, 1996). Under certain conditions, it has been found that the sharing is gratifying to the benefactor (de Waal et al., 2008). This begging cue and the sharing-based division problem could be one in the same. The instinct which follows the begging cue may be homologous to the instinct which is elicited when present-day humans give money to beggars (de Waal, 1996), allows for "... food *division* [italics added]..." (de Waal, 1996, p. 146), and the sharing of items by children to solve mathematical division problems.

### *Equality and Egalitarianism*

In order for sharing to take place, one must have the ability to quantify or measure an item against another item. For example, one must be able to compare the amount of one piece of meat to another piece and determine if the pieces are equal to one another. It is believed that equality and other egalitarian instincts came about in our human lineage as early humans were faced with environmental pressures. These pressures influenced the development of an instinct, which enables one to produce and quantify equal shares of a resource (Dunbar, 1997; Erdal & Whiten, 1994, 1997); the most important aspect of which was the ability to share food (de Waal, 1996; Ridley, 1996). In this society, an individual would have worked to get at least a share that is acceptable, equitable (Erdal & Whiten, 1994), and fair (Trivers, 1971). This instinct would have provided an advantage to certain individuals within the population and would have been selected for; therefore, increasing the population's fitness. This is because the individual who has the ability to detect whether one gets more than he is one who can realize if he has been cheated. If one does not have this ability, one may starve if food is scarce, while others would prosper from receiving larger, unequal shares. Numerous psychological studies have exhibited a pancultural human ability to detect when one has received an unequal share. Typical responses to another who has received a larger, unequal share are envy and jealousy (Tanaka, 1980).

Human infants, rhesus monkeys, capuchins, and chimpanzees can quantify and compare continuous and discontinuous objects (Beran, 2004; Judge, Evans, & Vyas, 2005; vanMarle, Aw, McCrink, & Santos, 2006). Capuchins can compare and quantify continuous and discontinuous foods, and can choose the greater amount of food (vanMarle et al., 2006; vanMarle & Wynn, 2002). Interestingly, capuchins were found to compare and quantify continuous foods (e.g., yogurt raisins) as well as discontinuous foods (e.g., banana puree) (vanMarle et al., 2006). This is significant because it signals that human ancestors may have been able to quantify or measure an item against another item, regardless if it was continuous or discontinuous. Human ancestors who killed a large animal (a continuous food) could use the ability to tear the carcass into mostly equivalent pieces, and deal the meat to others (a discontinuous food).

### **Discussion and Implications**

Research has shown that very young children have the ability to solve difficult division problems before formal school-based instruction. This is accomplished by solving sharing-based division word problems by an interesting act known as dealing. It has been asserted that dealing has its origins in sharing (Correa et al., 1998; Dickson, Brown, & Gibson, 1984) and that the ability to quantify food may have shaped certain mathematical abilities (Ansari, 2008); however, a thorough

explanation for this ability has never been formulated. The theory set forth in this manuscript is that children's ability to solve sharing-based division problems is due to an evolved psychological structure expressed as a dealing action. This structure may have been shaped by an ancient ecology that pressured human ancestors to cooperatively share food in order to survive, and contributed to the emergence of formal mathematical division.

The scenario for the origin of this ability and its contribution to formal mathematical division probably has an ancient phylogenetic history, and may have preceded the split of Old World monkeys and apes 25 million years ago. It may have unraveled as follows: a group of human-capuchins ancestors existed in a nonegalitarian social structure in an environment with a scarce food supply. Despite the social structure, the individuals had the ability to hunt and kill large vertebrates and distribute meat like extant capuchin monkeys. This organism, like many extant organisms, also had primitive sensitivity to numerosities, could reduce multiple complex inputs to simple numerical relationships, and could compare and determine which of two collections of meat were greater. Unlike extant nonhuman primates who have little need to share food, the scarce food supply created an ecological pressure in which sharing of meat was advantageous to the population. This is believed to be true because the ability to share food probably would have never come about if the pressure was not present, and explains why most primates do not share food as often as humans (de Waal, 1996; McGrew, 1992). A complex interplay between a selfish need for meat and other resources versus an egalitarian instinct, which included the ability to share, were about to be fought (Erdal & Whiten, 1994). The result was the present competing human psychological instincts which include the motivation to dominate others versus not to dominate, and to share food versus not to share (Erdal & Whiten, 1994). These same instincts, although to lesser degrees, seem to be present in capuchins (de Waal, 2000; de Waal & Davis, 2003).

An action for the sharing of food to others, which is found in human, capuchins, and other nonhuman primates, was selected for and shaped an action known as dealing. The dealing of food to others was necessary due to the multiple individuals that had to be fed and provided the most parsimonious manner in which to equally distribute food. Once again, this ability seems to be found in capuchin monkeys but to a lesser degree. Individuals who had the ability to share food through dealing only shared with others who had this same egalitarian instinct. This is because these individuals had an ability in which one could recognize fairness and whether one was being cheated or not. This improved the individual group member's fitness because of the precarious nature of hunting. Those group members who did not have this ability either starved when the food supply was scarce or when and if they continually returned from a hunt without food. As encephalization continued, the internalization of dealing was possible and sharing-based division problems became a mental model. Present day *Homo sapiens*

continued to use sharing through dealing, which quantitatively represented the behavior of a deal, was co-opted by culture (i.e., exaptation), and was conceptually linked to the discipline of mathematics. The two types of sharing situation, which are an inverse to multiplication, were further reduced into the two multiplicative problems types known as partitive and quotitive division. These became the problems that children are presently successful at solving. From here, the problem types were further reduced by mathematicians to a theorem that could be solved with the traditional division algorithm. The mathematical symbols for division were created along with division-related terminology (i.e, dividend, divisor, quotient, etc.).

Although the evolved psychological structure expressed as a dealing action is phylogenetically ancient, it is probably not as ancient as the number structure. This is because certain number structure abilities are necessary to fairly deal food to multiple individuals in a parsimonious manner. This includes the ability to: a) individuate the numerosities in a collection, b) instantaneously count or estimate numerosities in a collection, c) explain which of two collections is larger, d) enumerate collections, and e) recognize changes in numerosities within collections. However, one does not need the ability to match the numerosities of one collection based on one-to-one correspondences between count-words and the objects being counted. This means that the evolved psychological structure expressed as a dealing action could not have risen independently of the number structure, but was probably built upon it. In fact, it seems that certain aspects of the number structure are more phylogenetically ancient than others (Coolidge & Overmann, 2012).

There are several limitations to this scenario and the theory put forth in this manuscript. This first, and most obvious alternative explanation, is that the evolved psychological structure expressed as a dealing action could be nothing more than a vestigial structure which once held a purpose other than sharing food. It could be, say, useful for dealing tools. The second is that the action may be a random behavior with no purpose for survival all; it may simply be a mutation or a byproduct of another structure. For example, bone tissue is white, but the color isn't important for survival. Lastly, and although highly unlikely, the evolved psychological structure expressed as dealing and the number structure may be products of convergent evolution. These structures' abilities may have come about multiple times throughout deep time, and are simply interpreted as being homologous (Ansari, 2008).

Although children's ability to solve sharing-based division problems seems to be mathematically special, there is strong evidence for an evolutionary precursor for the operation of addition as well. The operation of addition requires mentally joining quantitative representations of numerosities. Infants seem to have this ability; many nonhuman primates do as well (Cantlon & Brannon, 2007). Evidence of this ability has been found in chimpanzees (Boysen & Berntson, 1989; Cantlon & Brannon, 2007), orangutans (Call, 2000), and rhesus monkeys (Flombaum,

Junge, & Hauser, 2005). Cantlon and Brannon (2007) concluded that addition performed by chimpanzees is similar to that of humans, and that it is part of a shared evolutionary primitive system. Once again, it seems that this operation is not as ancient as the number structure because the number structure's abilities are necessary to perform addition.

An implication of this theory deals with the teaching of division in elementary schools. This theory furthers the argument that mathematics should be introduced with an understanding of students' prior knowledge (see National Council of Teachers of Mathematics [NCTM], 2000; National Research Council, 2000). This means that students should be placed in a learning situation in which they can use this ability, and the teacher builds upon it, much like culture built upon the evolved psychological structure for dealing and the number structure. This is in opposition of how mathematical division is often introduced in schools today, in which the student is treated as a blank slate and explicitly taught the traditional algorithm with symbolic notation. Recent documents in US mathematics education have expressed the importance of prior knowledge in the teaching of mathematics in elementary school (see NCTM, 2000; National Mathematics Advisory Panel [NMAP], 2008). *Principles and Standards for School Mathematics*, the leading resource and guide for those who make decisions about the teaching and learning of K-12 mathematics education in the United States, states that children "... must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM, 2000)". *The Final Report of the National Mathematics Advisory Panel* (NMAP, 2008), a document, which outlines recommendations for stakeholders in US schools, states that "... [m]ost children acquire considerable knowledge of numbers and other aspects of mathematics before they enter kindergarten ... [and the]... mathematical knowledge that ... [children]...bring to school is related to their mathematics learning ..." (NMAP, 2008, p. xviii). Therefore, the evolved psychological structure for sharing/dealing can be used as a launching point to introduce and teach mathematical division.

An example lesson for the introduction of division for young children would appear as follows: The teacher passes out bags with three toy bear manipulatives and multiple small marshmallows to each student. The teacher asks the students to take out the bears and marshmallows, and to place the bears so that they face the student. The teacher tells the students that they will solve some sharing problems, and allows the students to work together to solve the problems. The teacher asks a partitive division problem, "You have six marshmallows and want to evenly share them among the three bears. How many marshmallows does each bear get?" The teacher provides time for the students to solve the problem. After the students finish solving the problem, they present their strategies and answers to the class. The teacher provides another more challenging problem by increasing the number of bears (divisor) or marshmallows (dividend). When the students finish multiple problems, the teacher introduces a formal mathematical number sentence, which

describes the operation. For example,  $6 \div 3 = 2$ . This type of explicit instruction would help students formalize the dealing action into a mathematical model.

There are multiple empirical questions that may be asked and recommendations for future study. If the theory set forth in this manuscript is correct, then it may be possible to empirically test the evolved psychological structure for dealing in young children to understand the food sharing habits, ecology, and sociology of early humans. This, of course, may be as misled as the ontogeny recapitulates phylogeny theory, yet it may lead to insights on food sharing questions that have eluded anthropologists and primatologists. Specific questions include: If the evolved psychological structure for dealing was formed around the sharing of meat, then would children be more proficient at dealing meat than other items (e.g., counters, coins, and marbles)? Can children deal to inanimate objects (i.e. bags, containers) as well as they do to individuals or anthropomorphized items (e.g., toy bears, dolls). Do children share the food at hand when others beg and do they use the dealing action? Do children use dealing in a naturalistic setting (e.g., outside of the classroom or lab), and if so, what is the cue to deal? Are children better at solving sharing-based division problems or problems which would use other operations (e.g., addition or multiplication)?

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## **Evoluirana psihološka struktura za podjelu, razmjenu hrane i matematičko dijeljenje**

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### **Sažetak**

Istraživanja su pokazala da djeca i prije početka formalnoga obrazovanja mogu rješavati zahtjevne matematičke zadatke dijeljenja. To se potvrđuje i prilikom rješavanja matematičkih problemskih zadataka razmjene pomoću postupka nazvanoga "podjela". Teorija prikazana u ovom radu pretpostavlja da dječja sposobnost rješavanja takvih problema proizlazi iz evoluirane psihološke strukture odgovorne za postupak podjele. Ta je struktura mogla biti oblikovana ekološkim uvjetima koji su tijekom evolucijske prošlosti kod naših predaka doveli do kooperativnoga dijeljenja hrane, što je pridonijelo nastanku sposobnosti matematičkoga dijeljenja. U prilog teoriji ide postojanje specijaliziranoga postupka podjele prilikom rješavanja problemskih zadataka razmjene, kulturalno univerzalna dječja sposobnost rješavanja takvih problema, izostanak svjesnosti prilikom rješavanja takvih problema, razvoj od podjele do mentalnoga modela, zatim evidencija o postojanju evoluiranih kvantitativnih sposobnosti kod ljudi i životinja, postupci razmjene hrane kod majmuna kapucina te sposobnost ravnopravne podjele. U raspravi je prikazan moguć scenarij razvoja opisane evoluirane strukture, ilustrirano je kako struktura može biti iskorištena prilikom osnovnoškolskoga poučavanja te su navedena empirijska pitanja koja mogu usmjeriti daljnja istraživanja s ciljem boljega razumijevanja postupaka dijeljenja hrane, ekologije i sociologije naših predaka.

**Ključne riječi:** matematičko dijeljenje, evoluirana psihološka struktura, razmjena hrane, kvantitativne sposobnosti, majmuni kapucini, podjela, rješavanje problema

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